

$\Lambda_c \rightarrow \Lambda \ell^+ \nu_\ell$ form factors and decay rates from lattice QCD with physical quark masses

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The decays $\Lambda_c \rightarrow \Lambda \ell^+ \nu_\ell$, where $\ell = e, \mu$, are the most important baryonic $c \rightarrow s \ell^+ \nu_\ell$ transitions. These processes can be used to determine the Cabibbo-Kobayashi-Maskawa quark mixing matrix element $|V_{cs}|$, to normalize branching fraction measurements of other heavy-baryon decays, and to test theoretical methods for heavy-baryon decay form factors. Here, the first lattice QCD calculation of the form factors that govern $\Lambda_c \rightarrow \Lambda \ell^+ \nu_\ell$ decays is reported. The calculation was performed with $2 + 1$ flavors of dynamical domain-wall fermions, using two different lattice spacings and a range of quark masses, including one ensemble with approximately physical quark masses corresponding to a pion mass of 139(2) MeV. The form factors are extrapolated to the continuum limit and are parametrized using model-independent z -expansions. The resulting predictions for the $\Lambda_c \rightarrow \Lambda e^+ \nu_e$ and $\Lambda_c \rightarrow \Lambda \mu^+ \nu_\mu$ decay rates divided by $|V_{cs}|^2$ are $0.2007(71)(74) \text{ ps}^{-1}$ and $0.1945(69)(72) \text{ ps}^{-1}$, respectively, where the two uncertainties are statistical and systematic. Taking $|V_{cs}|$ from a global CKM fit and the Λ_c lifetime from experiments, this translates to branching fractions of $\mathcal{B}(\Lambda_c \rightarrow \Lambda e^+ \nu_e) = 0.0380(19)_{\text{LQCD}}(11)_{\tau_{\Lambda_c}}$ and $\mathcal{B}(\Lambda_c \rightarrow \Lambda \mu^+ \nu_\mu) = 0.0369(19)_{\text{LQCD}}(11)_{\tau_{\Lambda_c}}$. These results are consistent with, and two times more precise than, the measurements performed recently by the BES III Collaboration. Using instead the measured branching fractions together with the lattice calculation to determine the CKM matrix element gives $|V_{cs}| = 0.949(24)_{\text{LQCD}}(14)_{\tau_{\Lambda_c}}(49)_{\mathcal{B}}$.

Precision studies of processes in which heavy bottom or charm quarks decay to lighter quarks play an important role in testing the Standard Model of elementary particle physics. While most of these analyses are being performed using B and D mesons, decays of Λ_b and Λ_c baryons can provide valuable additional information. Two examples that shed new light on puzzles posed by mesonic decays are the determination of the ratio of CKM matrix elements $|V_{ub}/V_{cb}|$ from the $\Lambda_b \rightarrow p \mu^- \bar{\nu}_\mu$ and $\Lambda_b \rightarrow \Lambda_c \mu^- \bar{\nu}_\mu$ decay rates [1], and an analysis of the rare $b \rightarrow s \mu^+ \mu^-$ transition using $\Lambda_b \rightarrow \Lambda \mu^+ \mu^-$ [2]. Both studies rely on nonperturbative lattice QCD calculations of form factors describing the baryonic matrix elements of the underlying quark currents [3, 4].

This letter focuses on the charmed-baryon decays $\Lambda_c \rightarrow \Lambda \ell^+ \nu_\ell$ ($\ell = e, \mu$), whose rates are proportional to $|V_{cs}|^2$ in the Standard Model. Previous determinations of this CKM matrix element are

$$|V_{cs}| = \begin{cases} 1.008(5)(16) & \text{from } D_s \rightarrow \ell^+ \nu_\ell \text{ [5, 6],} \\ 0.975(25)(7) & \text{from } D \rightarrow K \ell^+ \nu_\ell \text{ [6, 7],} \\ 0.97344(15) & \text{indirect, CKM unitarity [8].} \end{cases} \quad (1)$$

The motivations for studying $\Lambda_c \rightarrow \Lambda \ell^+ \nu_\ell$ include the following:

1. Taking the precisely determined value of $|V_{cs}|$ from CKM unitarity, a comparison between calculated and measured $\Lambda_c \rightarrow \Lambda \ell^+ \nu_\ell$ decay rates provides a stringent test of the methods used to compute the heavy-baryon decay form factors.
2. Combining the $\Lambda_c \rightarrow \Lambda \ell^+ \nu_\ell$ decay rates from experiment with a lattice QCD calculation of the $\Lambda_c \rightarrow \Lambda$ form factors gives a new direct determination of $|V_{cs}|$ and new constraints on physics beyond

the Standard Model (see, e.g., Ref. [9] for a recent discussion of new physics in $c \rightarrow s \ell^+ \nu_\ell$ transitions).

3. If the $\Lambda_c \rightarrow \Lambda \ell^+ \nu_\ell$ decay rates are known precisely, from experiment or lattice QCD, these modes can be used as normalization modes in measurements of a wide range of other charm and bottom baryon decays [10].

The most precise measurements of the $\Lambda_c \rightarrow \Lambda \ell^+ \nu_\ell$ branching fractions (decay rates times the Λ_c lifetime) to date have recently been reported by the BES III Collaboration [11, 12],

$$\mathcal{B}(\Lambda_c \rightarrow \Lambda \ell^+ \nu_\ell) = \begin{cases} 0.0363(38)(20), & \ell = e, \\ 0.0349(46)(27), & \ell = \mu. \end{cases} \quad (2)$$

In the Standard Model, the decay rates depend on six form factors that parametrize the matrix elements $\langle \Lambda(p') | \bar{s} \gamma^\mu c | \Lambda_c(p) \rangle$ and $\langle \Lambda(p') | \bar{s} \gamma^\mu \gamma_5 c | \Lambda_c(p) \rangle$ as functions of $q^2 = (p - p')^2$. These form factors have previously been estimated using quark models and sum rules [13–29], giving branching fractions that vary substantially depending on the model assumptions. In the following, the first lattice QCD determination of the $\Lambda_c \rightarrow \Lambda$ form factors is reported. The calculation uses state-of-the-art methods and gives predictions for the $\Lambda_c \rightarrow \Lambda \ell^+ \nu_\ell$ decay rates with total uncertainties that are smaller than the experimental uncertainties in Eq. (2) by a factor of two.

This work is based on gauge field configurations generated by the RBC and UKQCD collaborations with $2 + 1$ flavors of dynamical domain-wall fermions [30, 31]. The data sets used here are listed in Table I, and match those in Refs. [3] and [4], except for the addition of a new ensemble (denoted as CP) with $m_\pi = 139(2) \text{ MeV}$, and the

Set	β	$N_s^3 \times N_t$	$am_{u,d}^{(\text{sea})}$	$am_s^{(\text{sea})}$	a [fm]	$am_{u,d}^{(\text{val})}$	$m_\pi^{(\text{val})}$ [MeV]	$am_s^{(\text{val})}$	$m_{\eta_s}^{(\text{val})}$ [MeV]	N_{samples}
CP	2.13	$48^3 \times 96$	0.00078	0.0362	0.1142(15)	0.00078	139(2)	0.0362	693(9)	2560 sl, 80 ex
C54	2.13	$24^3 \times 64$	0.005	0.04	0.1119(17)	0.005	336(5)	0.04	761(12)	2782
C53	2.13	$24^3 \times 64$	0.005	0.04	0.1119(17)	0.005	336(5)	0.03	665(10)	1205
F43	2.25	$32^3 \times 64$	0.004	0.03	0.0849(12)	0.004	295(4)	0.03	747(10)	1917
F63	2.25	$32^3 \times 64$	0.006	0.03	0.0848(17)	0.006	352(7)	0.03	749(14)	2782

TABLE I. Parameters of the lattice gauge field ensembles and u , d , s quark propagators [30, 31]. The lattice spacings given here were determined using the $\Upsilon(2S) - \Upsilon(1S)$ splitting [32]. The η_s is an artificial pseudoscalar $s\bar{s}$ meson used to tune the strange-quark mass [33]; at the physical point, one has $m_{\eta_s}^{(\text{phys})} = 689.3(1.2)$ MeV [34]. On the CP ensemble, *all-mode-averaging* [35] with 64 sloppy (sl) and 2 exact (ex) samples per gauge configuration was used for the computation of the quark propagators.

Set	am_{Λ_c}	am_Λ	am_{D_s}	am_D
CP	1.3194(36)	0.6483(33)	1.12902(39)	1.0720(12)
C54	1.3706(40)	0.7348(30)	1.13156(49)	1.0763(13)
C53	1.3647(60)	0.7096(47)	1.11550(59)	1.0763(13)
F43	1.0185(67)	0.5354(29)	0.85447(47)	0.81185(91)
F63	1.0314(40)	0.5514(23)	0.85639(33)	0.81722(56)

TABLE II. Hadron masses in lattice units obtained from exponential fits to two-point functions.

Parameter	Coarse lattice	Fine lattice
$\rho_{V^0} = \rho_{A^0}$	1.00274(49)	1.001949(85)
$\rho_{V^j} = \rho_{A^j}$	0.99475(62)	0.99675(68)
$c_{V^0}^R = c_{A^0}^R$	0.0402(88)	0.0353(92)
$c_{V^0}^L = c_{A^0}^L$	-0.0048(48)	-0.0027(28)
$c_{V^j}^R = c_{A^j}^R$	0.0346(51)	0.0283(43)
$c_{V^j}^L = c_{A^j}^L$	0.00012(26)	0.00040(42)
$d_{V^j}^R = -d_{A^j}^R$	-0.0041(41)	-0.0039(39)
$d_{V^j}^L = -d_{A^j}^L$	0.0021(21)	0.0026(26)

removal of the previous “partially quenched” C14, C24, F23 data sets which had $am_{u,d}^{(\text{val})} < am_{u,d}^{(\text{sea})}$. Adding the CP ensemble significantly aids in the extrapolation of the form factors to the physical point, and removing the partially quenched data sets reduces finite-volume effects.

The charm quark is implemented using an anisotropic clover action, with parameters tuned to produce the correct J/ψ relativistic dispersion relation as quantified by the “speed of light”, c , and the correct spin-averaged mass $\bar{m} = \frac{3}{4}m_{J/\psi} + \frac{1}{4}m_{\eta_c}$ [36]. On the new CP ensemble, the same bare parameters as tuned on the coarse $24^3 \times 64$ lattice yield $c = 0.9970(27)$ and $\bar{m} = 3019(40)$ MeV, consistent with the experimental value of 3068.5(0.1) MeV [37], and were therefore used on this ensemble as well. The Λ_c , Λ , D_s , and D masses obtained from the different data sets are listed in Table II.

The renormalization of the $c \rightarrow s$ vector and axial vector currents is performed using the *mostly non-perturbative* method [38, 39] as in Eqs. (18)-(21) of Ref. [3] (with the replacements $b \rightarrow c$, $q \rightarrow s$). The nonperturbative coefficients used here on the coarse $48^3 \times 96$, coarse $24^3 \times 64$, and fine $32^3 \times 64$ lattices are $Z_V^{(ss)} = 0.71076(25), 0.71273(26), 0.74404(181)$ [30] and $Z_V^{(cc)} = 1.35899(22), 1.35725(23), 1.18321(14)$; the residual matching coefficients and $\mathcal{O}(a)$ -improvement coefficients were computed in tadpole-improved one-loop lattice perturbation theory [40, 41] and are given in Table III.

The $\Lambda_c \rightarrow \Lambda$ form factors are defined as in Eqs. (1) and (2) of Ref. [4] (with b replaced by c), and were extracted from ratios of three-point and two-point correlation func-

TABLE III. Residual matching and improvement coefficients for the $c \rightarrow s$ vector and axial vector currents, computed using automated lattice perturbation theory [40, 41]. The notation is the same as in Eqs. (18)-(21) of Ref. [3].

tions using the same methods as in [3, 4]. This involves extrapolations to infinite source-sink separation to isolate the ground-state contributions, which are performed jointly for all data sets at matching Λ momenta [3, 4]. These momenta, $|\mathbf{p}'|^2$, were set to 1, 2, 3, and 4 times $(2\pi/L)^2$ on all data sets except CP. For the latter, the values 4, 8, 12, and 16 times $(2\pi/L)^2$ were used because $L = N_s a$ is twice as large. The ranges of source-sink separations were $t/a = 4 \dots 15$ on the coarse lattices and $t/a = 5 \dots 17$ on the fine lattices; full $\mathcal{O}(a)$ -improvement of the currents was performed for all source-sink separations (instead of just a subset as in Refs. [3, 4]). Examples for the ratios and $t \rightarrow \infty$ extrapolations are shown in Fig. 1.

The ground-state form factors obtained in this way for the different data sets and different discrete momenta are shown as the data points in Fig. 2. To obtain parametrizations of the form factors in the physical limit ($a = 0$, $m_\pi = m_{\pi, \text{phys}}$, $m_{\eta_s} = m_{\eta_s, \text{phys}}$), fits were then performed using z -expansions [42] modified with additional terms to describe the dependence on a , m_π , and m_{η_s} . In the physical limit, the fit functions reduce to the form

$$f(q^2) = \frac{1}{1 - q^2/(m_{\text{pole}}^f)^2} \sum_{n=0}^{n_{\text{max}}} a_n^f [z(q^2)]^n, \quad (3)$$

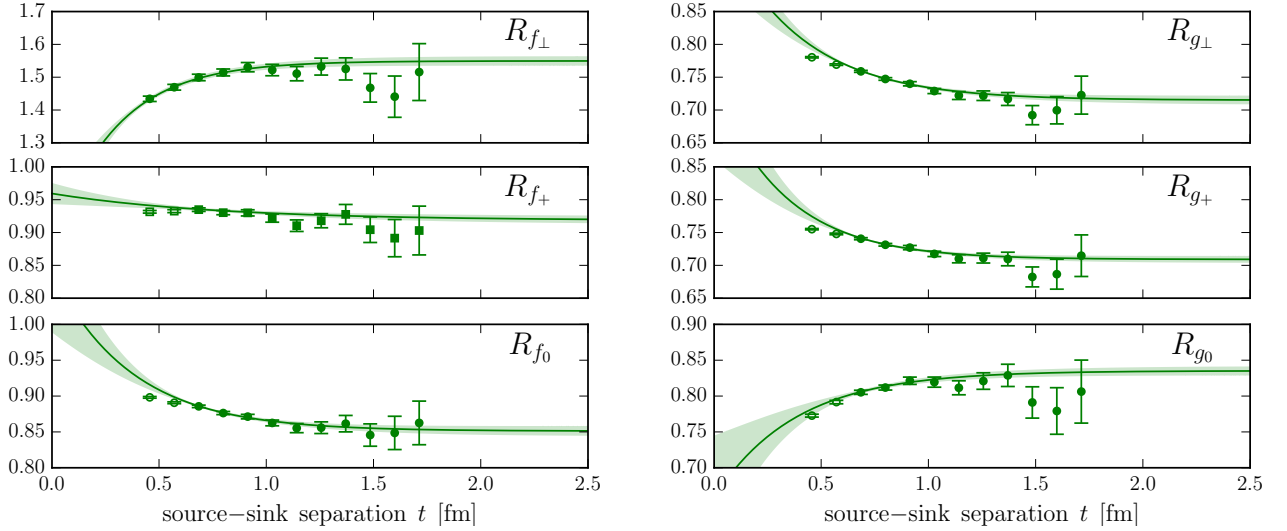


FIG. 1. Lattice results from the \mathbb{CP} ensemble for the ratios $R_f(|\mathbf{p}'|, t)$, defined as in Eqs. (52-54) and (58-60) of Ref. [3], at $|\mathbf{p}'|^2 = 4(2\pi/L)^2$. Also shown are the fits that are used to extract the ground-state form factors $f(|\mathbf{p}'|) = \lim_{t \rightarrow \infty} R_f(|\mathbf{p}'|, t)$ [3, 4]. Data points at the smallest separations that are plotted with open symbols are excluded from the fit to reduce contamination from higher excited states.

where $z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$ with $t_0 = q_{\max}^2 = (m_{\Lambda_c} - m_\Lambda)^2$ and $t_+ = (m_D + m_K)^2$. The D_s meson pole masses are $m_{\text{pole}}^{f_+, f_\perp} = 2.112$ GeV, $m_{\text{pole}}^{f_0} = 2.318$ GeV, $m_{\text{pole}}^{g_+, g_\perp} = 2.460$ GeV, $m_{\text{pole}}^{g_0} = 1.968$ GeV [37], and to evaluate t_+ , the masses $m_D = 1.870$ GeV and $m_K = 494$ MeV are used. Following Refs. [3, 4], two separate fits were performed: a “nominal fit”, giving the central values and statistical uncertainties of the form factors, and a “higher-order fit”, used to compute systematic uncertainties according to Eqs. (50-56) of Ref. [4]. The nominal fit had the same form as Eq. (36) of Ref. [4], but with $n_{\max} = 2$ instead of $n_{\max} = 1$ (no prior constraints on any parameters were used in the nominal fit). The higher-order fit had the same form as in Eq. (39) of Ref. [4], but with $n_{\max} = 3$. In addition to the z^3 terms, this fit also includes terms of higher order in a , m_π , m_{η_s} , and was performed after modifying the data correlation matrix to include the uncertainties from the renormalization and $\mathcal{O}(a)$ -improvement coefficients, from finite-volume effects (1.0%, rescaled from Ref. [4] according to $e^{-\min[m_\pi L]}$), and from the missing isospin breaking/QED corrections (0.5%, 0.7%). The priors for the higher-order parameters were chosen as in Ref. [4], except that the coefficients a_2^f were left unconstrained and the priors for a_3^f were set to 0 ± 30 . The fit results for the parameters a_n^f that describe the form factors in the physical limit are given in Table IV, and the form factors are plotted in Fig. 2. The lattice results do not significantly constrain the z^3 terms (note that $z_{\max} \approx 0.08$), so that their uncertainties are governed by the priors.

The resulting Standard-Model predictions for the

	Nominal fit	Higher-order fit
$a_0^{f_\perp}$	1.30 ± 0.06	1.28 ± 0.07
$a_1^{f_\perp}$	-3.27 ± 1.18	-2.85 ± 1.34
$a_2^{f_\perp}$	7.16 ± 11.6	7.14 ± 12.2
$a_3^{f_\perp}$		-1.08 ± 30.0
$a_0^{f_+}$	0.81 ± 0.03	0.79 ± 0.04
$a_1^{f_+}$	-2.89 ± 0.52	-2.38 ± 0.61
$a_2^{f_+}$	7.82 ± 4.53	6.64 ± 6.07
$a_3^{f_+}$		-1.08 ± 29.8
$a_0^{f_0}$	0.77 ± 0.02	0.76 ± 0.03
$a_1^{f_0}$	-2.24 ± 0.51	-1.77 ± 0.58
$a_2^{f_0}$	5.38 ± 4.80	4.93 ± 6.28
$a_3^{f_0}$		-0.26 ± 29.8
$a_0^{g_\perp, g_+}$	0.68 ± 0.02	0.67 ± 0.02
$a_1^{g_\perp}$	-1.91 ± 0.35	-1.73 ± 0.54
$a_2^{g_\perp}$	6.24 ± 4.89	5.97 ± 6.64
$a_3^{g_\perp}$		-1.68 ± 29.8
$a_1^{g_+}$	-2.44 ± 0.25	-2.22 ± 0.35
$a_2^{g_+}$	13.7 ± 2.15	12.1 ± 4.43
$a_3^{g_+}$		12.9 ± 29.2
$a_0^{g_0}$	0.71 ± 0.03	0.72 ± 0.04
$a_1^{g_0}$	-2.86 ± 0.44	-2.80 ± 0.53
$a_2^{g_0}$	11.8 ± 2.47	11.7 ± 4.74
$a_3^{g_0}$		1.35 ± 29.4

TABLE IV. Results for the z -expansion parameters describing the form factors in the physical limit. Files containing the parameter values with more digits and the full covariance matrices are provided as supplemental material.

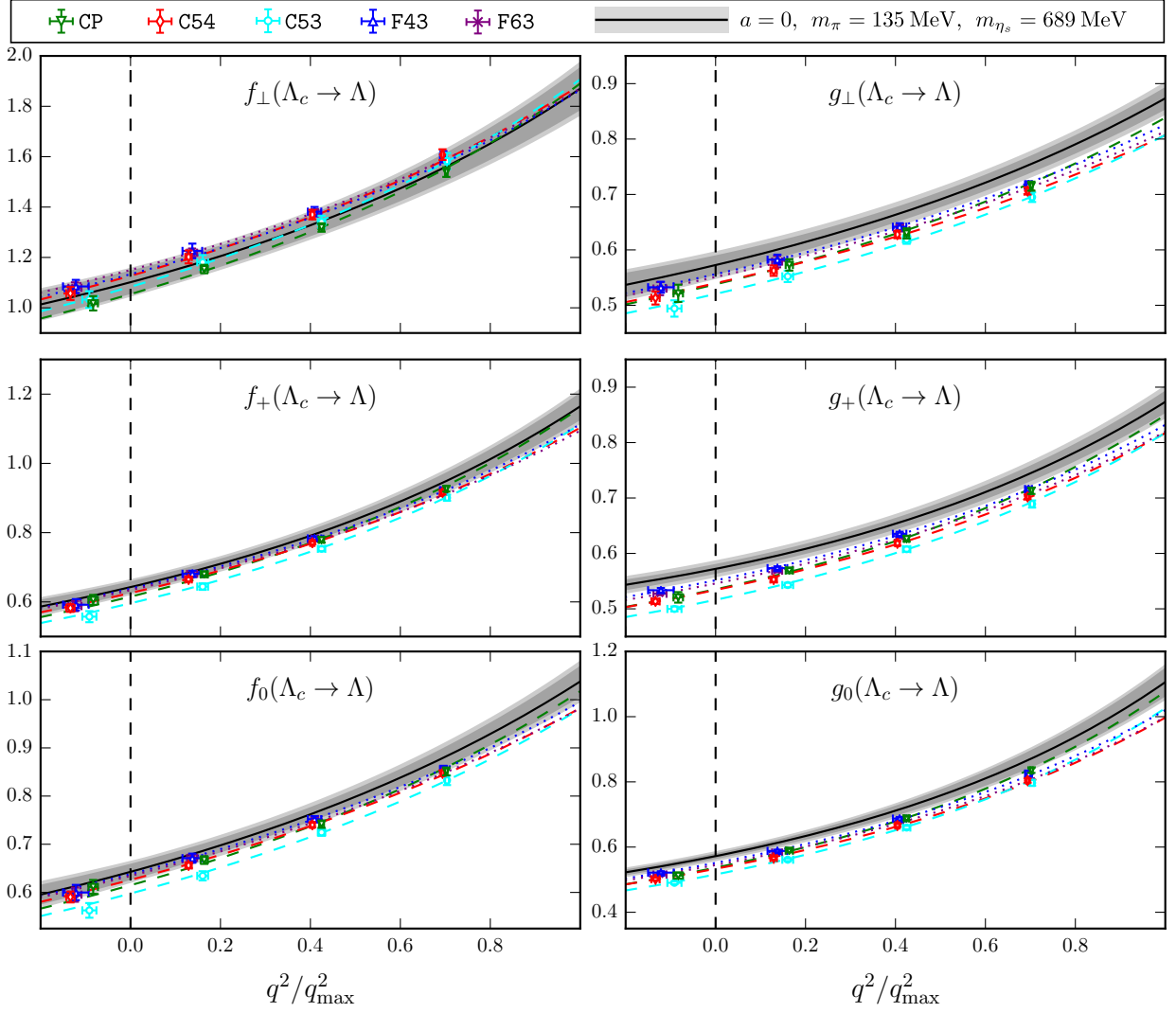


FIG. 2. Lattice QCD results for the $\Lambda_c \rightarrow \Lambda$ form factors, along with the modified z -expansion fits evaluated at the lattice parameters (dashed and dotted lines) and in the physical limit (solid lines, with statistical and total uncertainties indicated by the inner and outer bands).

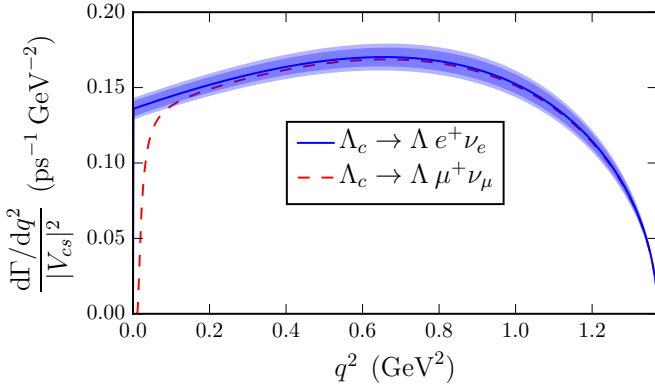


FIG. 3. Predictions for the $\Lambda_c \rightarrow \Lambda \ell^+ \nu_\ell$ differential decay rates (divided by $|V_{cs}|^2$) in the Standard Model. For clarity, the uncertainties are shown only for $\ell = e$; the inner and outer bands correspond to the statistical and total uncertainties.

$\Lambda_c \rightarrow \Lambda \ell^+ \nu_\ell$ differential decay rates, without the factor of $|V_{cs}|^2$, are shown in Fig. 3. The q^2 -integrated rates are

$$\frac{\Gamma(\Lambda_c \rightarrow \Lambda \ell^+ \nu_\ell)}{|V_{cs}|^2} = \begin{cases} 0.2007(71)(74) \text{ ps}^{-1}, & \ell = e, \\ 0.1945(69)(72) \text{ ps}^{-1}, & \ell = \mu, \end{cases} \quad (4)$$

where the two uncertainties are from the statistical and total systematic uncertainties in the form factors. Using the world average of Λ_c lifetime measurements, $\tau_{\Lambda_c} = 0.200(6) \text{ ps}$ [37], and $|V_{cs}| = 0.97344(15)$ from a CKM unitarity global fit [8] then yields the branching fractions

$$\mathcal{B}(\Lambda_c \rightarrow \Lambda \ell^+ \nu_\ell) = \begin{cases} 0.0380(19)_{\text{LQCD}(11)} \tau_{\Lambda_c}, & \ell = e, \\ 0.0369(19)_{\text{LQCD}(11)} \tau_{\Lambda_c}, & \ell = \mu, \end{cases} \quad (5)$$

where the uncertainties marked “LQCD” are the total form factor uncertainties from the lattice calculation.

These results are consistent with, and two times more precise than, the BES III measurements shown in Eq. (2). This is a valuable check of the lattice methods which were also used in Refs. [1–4].

Combining instead the BES III measurements (2) and $\tau_{\Lambda_c} = 0.200(6)$ ps with the results in Eq. (4) to determine $|V_{cs}|$ from $\Lambda_c \rightarrow \Lambda \ell^+ \nu_\ell$ gives

$$|V_{cs}| = \begin{cases} 0.951(24)_{\text{LQCD}(14)\tau_{\Lambda_c}(56)_B}, & \ell = e, \\ 0.947(24)_{\text{LQCD}(14)\tau_{\Lambda_c}(72)_B}, & \ell = \mu, \\ 0.949(24)_{\text{LQCD}(14)\tau_{\Lambda_c}(49)_B}, & \ell = e, \mu, \end{cases} \quad (6)$$

where the last line is the correlated average over $\ell = e, \mu$. This is the first determination of $|V_{cs}|$ from baryonic decays. The result is consistent with CKM unitarity, and the uncertainty can be reduced further with more precise measurements of the $\Lambda_c \rightarrow \Lambda \ell^+ \nu_\ell$ branching fractions.

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